4.2 Implicit Differentiation

Question Paper

Course	CIE AS Maths
Section	4. Differentiation
Topic	4.2 Implicit Differentiation
Difficulty	Very Hard

Time allowed: 60

Score: /46

Percentage: /100

Question 1

Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $e^{xy} + \ln(xy) = \operatorname{cosec}(x) + 4$

(ii) $4\cos(x^2 y) - 3e^{x^2 y} = 4e^y$

[5 marks]

Question 2

Find the gradient at the point where $x = -2$ and y is an integer on the curve with equation $x^2 y^2 - 5x = 22y$.

[5 marks]

Question 3

An ellipse has equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Find an expression for $\frac{dy}{dx}$ and hence show that the gradient of the ellipse at any point where it meets a line of the form $y = kx$ $(k \neq 0)$ is independent of x and y.

[4 marks]

Question 4

The curve C is described by the equation

$$\ln y + x^2 y^2 = 9.$$

Show that the tangents of the two points on C where $y = 1$ meet at the point $\left(0, \frac{37}{19}\right)$.

[5 marks]

Question 5a

The curve C is described by the equation

$$3x^2 + 2xy^3 + 16 = 0.$$

(a) Show that the normal to C at the point where $x = -4$ is parallel to the normal to C at the point where $x = 4$.

[4 marks]

Question 5b

(b) Find the distance between the y-axis intercepts of these two normals.

[4 marks]

Question 6

Find the stationary points and determine their nature for the curve with equation
$y^2 = 3x^2 - 2xy + 3.$

[8 marks]

Question 7

The curve C is defined by

$$e^{\sin(xy)} = 1 \qquad \{y > 0\}$$

Points A and B have coordinates $\left(\dfrac{\pi}{2}, 2\right)$ and $\left(-\dfrac{\pi}{2}, 2\right)$ respectively.

The tangents to C at points A and B intersect at the point P.
The tangent to C at point A intersects the x-axis at point Q.
The tangent to C at point B intersects the x-axis at point R.

Find the area of triangle PQR.

[8 marks]

Question 8

Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[a^{x^k}\right] = ka^{x^k}x^{k-1}\ln a$$

where a and k are constants.

[3 marks]